

FIG. 3

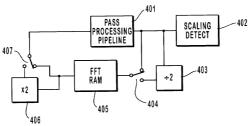


FIG. 4

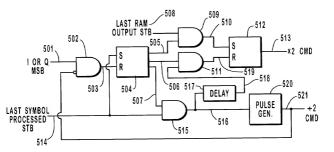


FIG. 5

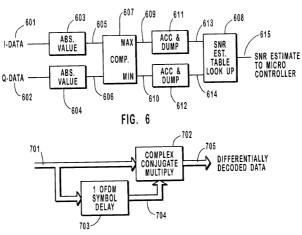


FIG. 7

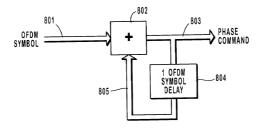
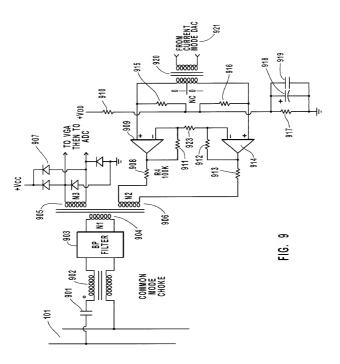
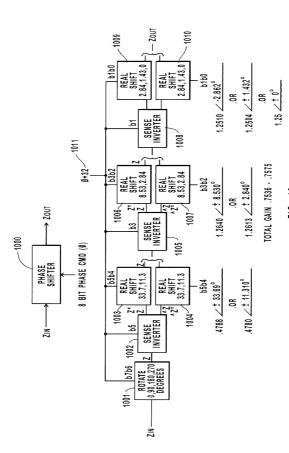
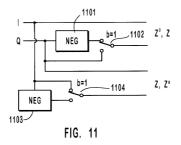


FIG. 8







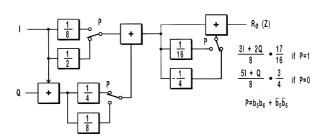


FIG. 12

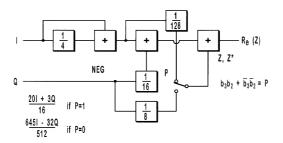


FIG. 13

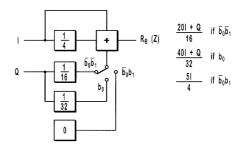
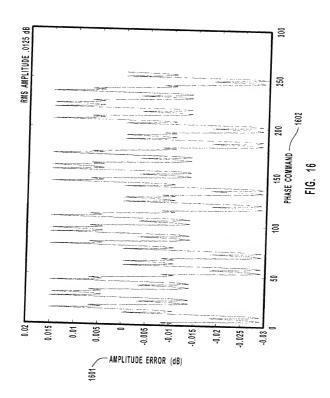
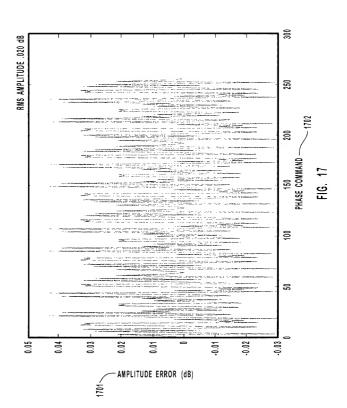
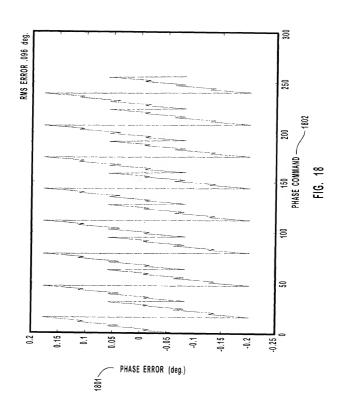


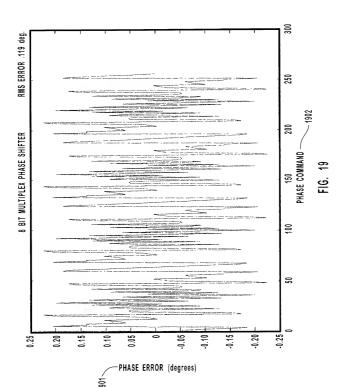
FIG. 14

```
function z = pshift l(z,th);
iz=real(z); qz=imag(z);
tha=2*(bitand(th+32,2.^{(0:7)})>0)-1; %make array of phase bits + 32
%0,90,180,270
if tha(7)==1;z=j*z;end;
if tha(8) = 1; z = -z; end;
% + -33.69.11.31
if tha(6)*tha(5) = -1;
       z=3/16*(5+tha(6)*i)*z;
else;
       z=17/64*(3+tha(6)*2*j)*z;
end:
% +- 8.53, 2.84
if tha(4)*tha(3) = =-1;
       z=(20*(1+1/128) + tha(4)*j)*z/32;
else;
       z=(20+tha(4)*3*i)*z/32;
end;
% -2.86 -1.43 0 1.43
if tha(1) = =1;
       z=(40+tha(2)*j)*z/32;
elseif tha(2)= = -1:
       z=(20-i)*z/16;
else:
       z=5*z/4:
end:
```









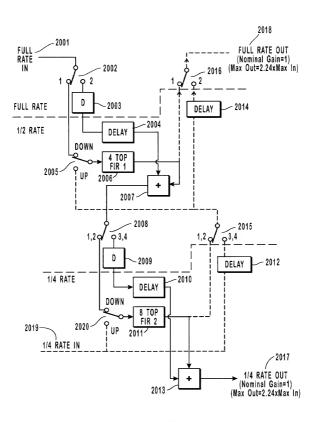


FIG. 20

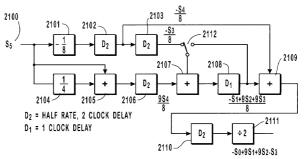


FIG. 21

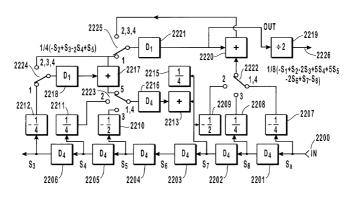


FIG. 22

17 / 68 Anatomy of the FFT

This is a 256 point base 2 example of an FFT (a fast dft)

$$k := 0.255$$

FFT index

$$x_i := rnd(1) + j \cdot rnd(1)$$

 $x_i := rnd(1) + j \cdot rnd(1)$ FFT dummy arguement

The discfrete forrier transform takes 256 ^2 operations

$$DFT_k := \sum_{i=0}^{255} x_i \cdot e^{j-2\pi} \frac{i \cdot k}{256}$$

If we combine terms from the first and 2nd half of the summation we have

$$\sum_{i=-0}^{127} \left(x_i + x_{i+128} \cdot e^{-j \cdot 2 \cdot \pi \cdot \frac{k}{2}} \right) \cdot e^{-j \cdot 2 \cdot \pi \cdot \frac{i \cdot k}{256}}$$

Note the term in paremphases runs over half of N and is only unique for k mod 2 SO: 11:=0.. 127 k1:=0.. 1

$$Xl_{i1,k1} := x_{i1} + x_{i1+128} \cdot e^{-(j-2\pi)\frac{k1}{2}}$$

Then

$$DFT1_{k} := \sum_{i=0}^{127} XI_{i, mod(k, 2)} e^{\int 2\pi \frac{i \cdot k}{256}} \sum_{k} \left(DFT_{k} - DFT1_{k} \right)^{2} = 0$$

$$\sum_{k} \left(\left| DFT_{k} - DFT1_{k} \right| \right)^{2} = 0$$

This operation only takes half the DFT steps by taking 128 steps to precompute X1 We can further reduce the computational load by doing the same thing again

$$\begin{split} &i1 := 0...63 & k1 := 0...3 \\ &X2_{i1.,k1} := &X1_{i1,\,\text{mod}(k1,\,2)} + &X1_{i1+64,\,\text{mod}(k1,\,2)} \cdot e \\ \end{split}$$

Then

$$DFT2_{k} := \sum_{i=0}^{63} X2_{i, mod(k, 4)} \cdot e^{\int_{0}^{2\pi} \frac{i \cdot k}{256}} \sum_{k} \left(|DFT_{k} - DFT2_{k}| \right)^{2} = 0$$

We repeat this operation a total of 8 times until i ranges only over zero We show one more trick. Instead of:

$$\begin{split} &i1 := 0...31 \qquad k1 := 0...7 \\ &X3_{i1,\,k1} := X2_{i1,\,mod(k1,4)} + X2_{i1+32,\,mod(k1,4)} \cdot e^{-(j-2\cdot\pi)\frac{k1}{8}} \end{split}$$

Use

$$k_1 := 0..3$$
 $t_{i1,k1} := X2_{i1+32,k1} \cdot e^{-(j-2\pi)\frac{k1}{8}}$

$$X3_{i1,k1} := X2_{i1,k1} + t_{i1,k1}$$
 $X3_{i1,k1+4} := X2_{i1,k1} - t_{i1,k1}$

It only requires half the phase shifts. This operation is called a **butterfly** Continuing

$$\begin{aligned} &\text{i1} := 0...15 & \text{k1} := 0...7 & & & & & & & & & & & & & & & & \\ &\text{i1} := 0...15 & & & & & & & & & & & & & \\ &\text{X4}_{i1,k1} := & & & & & & & & & & & \\ &\text{X4}_{i1,k1} := & & & & & & & & & \\ &\text{X4}_{i1,k1} := & & & & & & & & \\ &\text{X4}_{i1,k1} := & & & & & & & \\ &\text{X5}_{i1,k1} := & & & & & & \\ &\text{X6}_{i1,k1} := & & & & & \\ &\text{X7}_{i1,k1} := & & & & & \\ &\text{X8}_{i1,k1} := & & & & \\ &\text{X9}_{i1,k1} := & & & & \\ &\text{X9}_{i1,k1} := & & \\ &\text{X9}_{i1,k1} := & & & \\ &\text{X9}_{i1,k1} := & & \\ &\text{X9$$

$$\begin{split} & \text{i1} := 0...7 & \text{k1} := 0...15 & t_{i1,k1} := X4_{i1+8,k1} \cdot e^{-(j-2\pi)\frac{k1}{32}} \\ & X5_{i1,k1} := X4_{i1,k1} + t_{i1,k1} & X5_{i1,k1+16} := X4_{i1,k1} - t_{i1,k1} \end{split}$$

$$\begin{split} & \text{i1} := 0...3 & \text{k1} := 0...31 & \text{t}_{i1,k1} := X5_{i1,k1} \cdot e^{-(j-2\pi)\frac{kl}{64}} \\ & X6_{i1,k1} := X5_{i1,k1} + t_{i1,k1} & X6_{i1,k1+32} := X5_{i1,k1} - t_{i1,k1} \end{split}$$

and finally

$$k1 := 0.. 127 \qquad t_{0,k1} := X7_{1,k1} \cdot e^{-\left(j - 2\pi\right) \frac{k1}{256}}$$

$$X8_{k1} := X7_{0,k1} + t_{0,k1} \qquad X8_{k1+128} := X7_{0,k1} - t_{0,k1}$$

$$\sum_{k} \left(\left\| DFT_{k} - X8_{k} \right\| \right)^{2} = 0$$

$$FIG. 23b$$

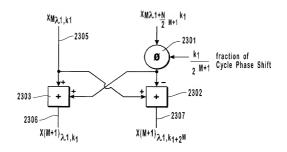


FIG. 23c

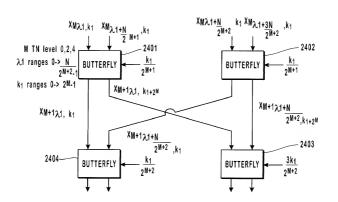
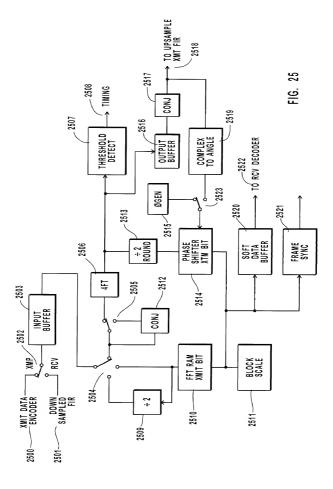
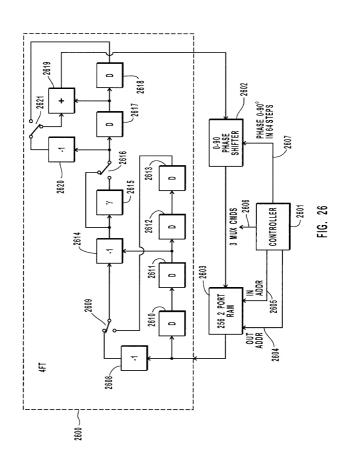
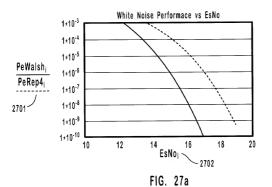
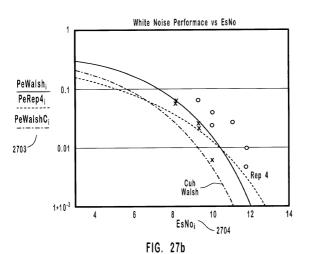


FIG. 24

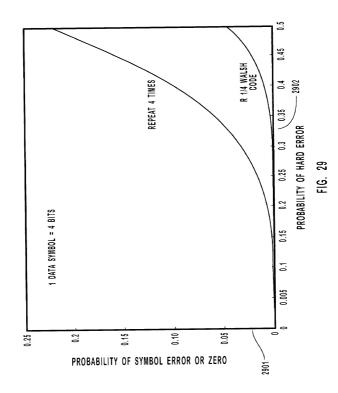


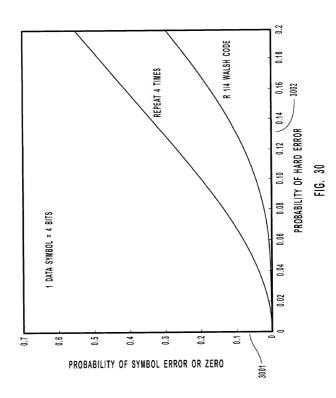






```
% finds likelihood of 16ary symbol error for nbe erasures
tr={1-1; 1 1}; tr=[tr-tr; tr tr]; tr=[tr-tr; tr tr]; tr=[tr-tr; tr tr];
v=zeros(1.16);
nsym=zeros(1,16);
while sum(v)<16;
       [a,n]=\max(tr*v');
       if n<16;
               nbe=sum(v==0);
               nsym(nbe)=nsym(nbe) +1;
       end;
       k=1;
       v(k) = -(v(k)-1); % -1 or 0 - erasures
       while v(k) = 0:
               k=k+1:
               v(k) = -(v(k)-1); % -1 or 0 – erasures
       end:
end;
pe=.005:.005:.5;
for k=1:100;
       we(k)=sum(pe(k).^{(1:16).*(1-pe(k)).^{(15:-1:0).*nsym});
end;
rhe=1-((1-pe).^4+4*pe.*(1-pe).^3).^4; % repeat sym hard error
rea=1-(1-pe.^4).64; % repeat sym erasure error
plot (pe,we,pe,rea)
```





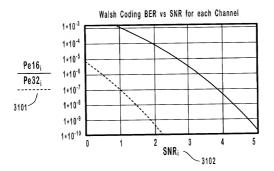


FIG. 31

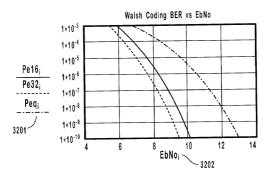
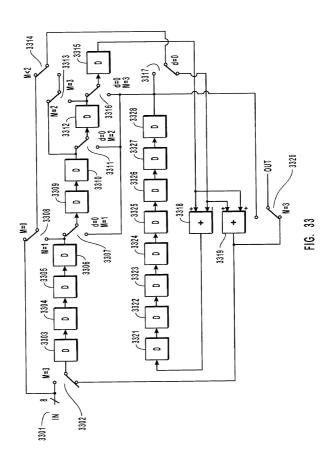


FIG. 32



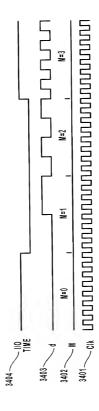


FIG. 34



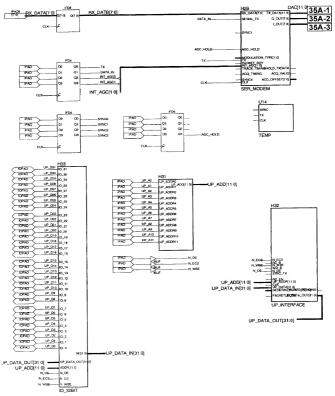
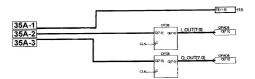


FIG. 35a-1





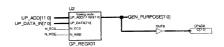
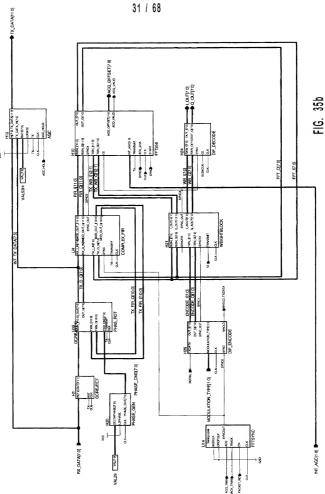
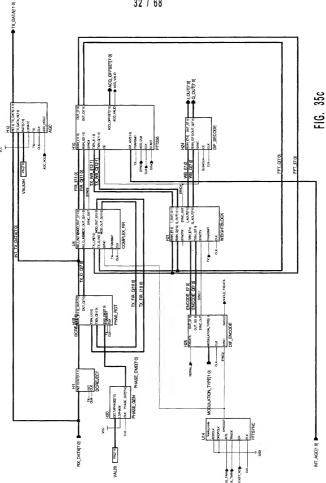


FIG. 35a-2





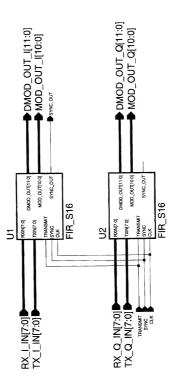
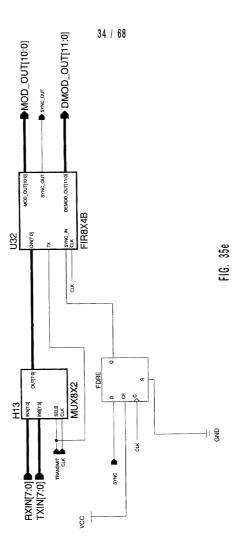


FIG. 35d





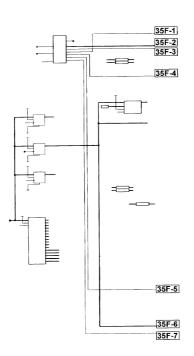


FIG. 35f-1

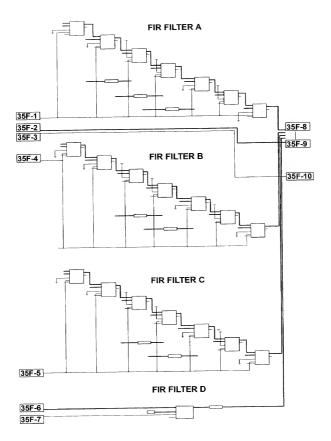


FIG. 35f-2

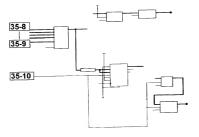


FIG. 35f-3

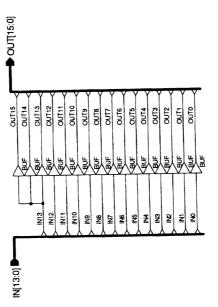
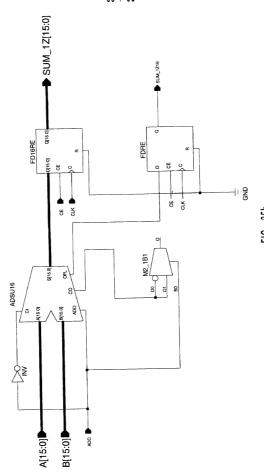
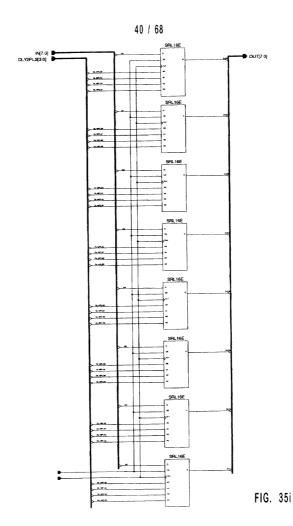


FIG. 35g





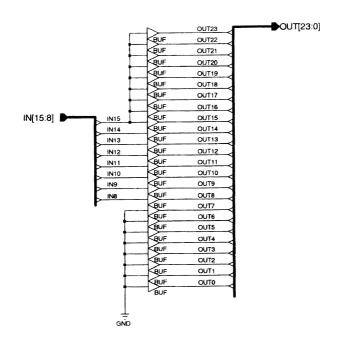
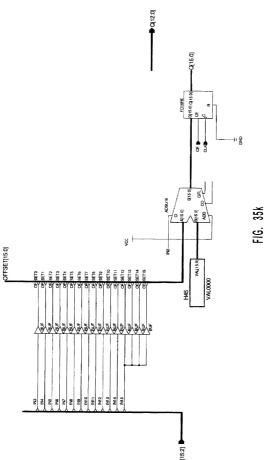
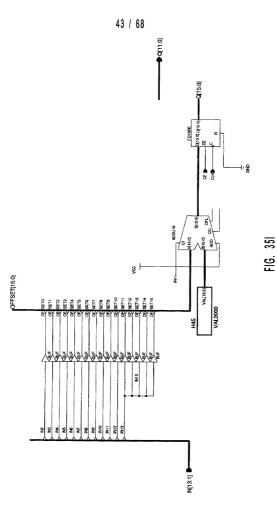


FIG. 35j





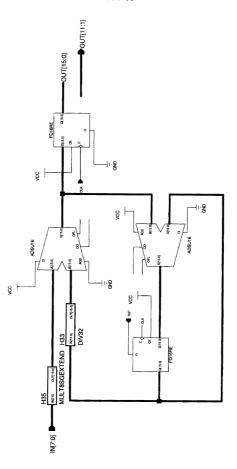
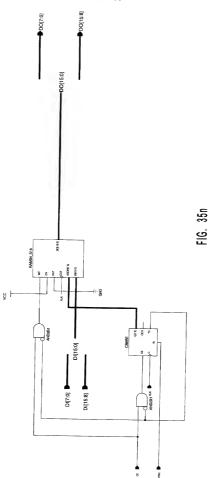


FIG. 35m



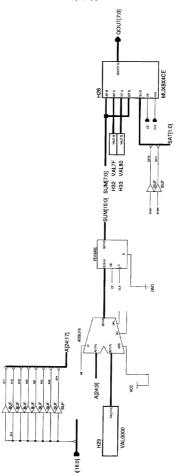


FIG. 350

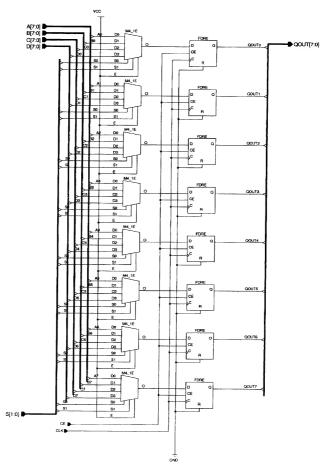


FIG. 35p

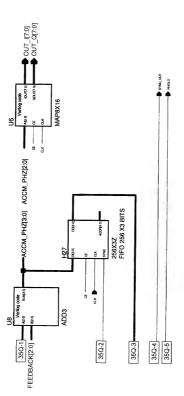


FIG. 35q-2

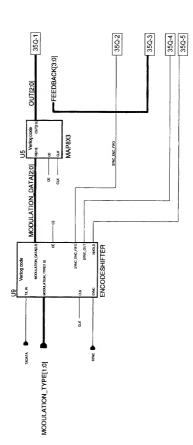


FIG. 35q-

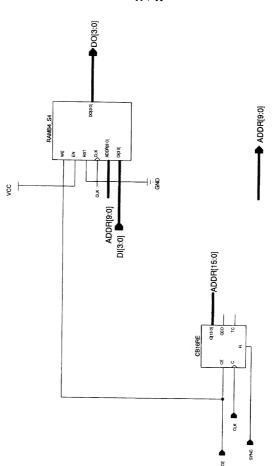
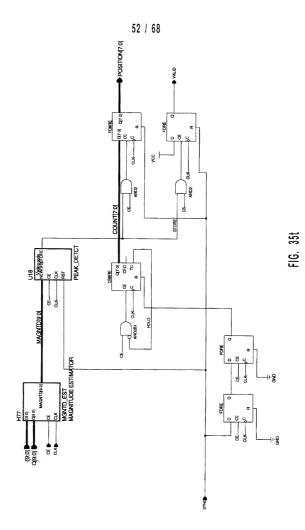


FIG. 35r

FIG. 35s



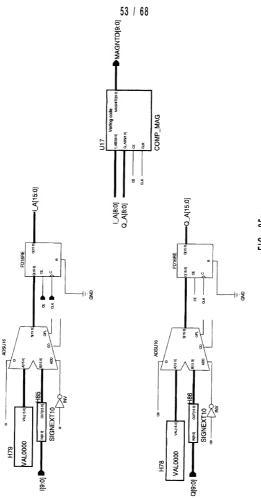


FIG. 35u

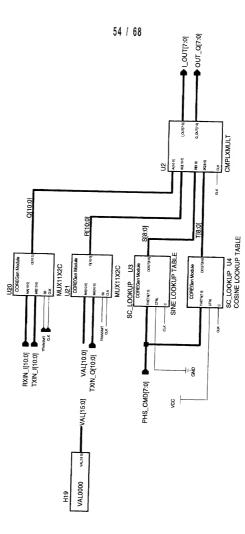


FIG. 35v

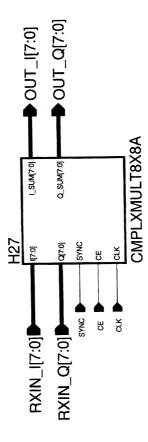
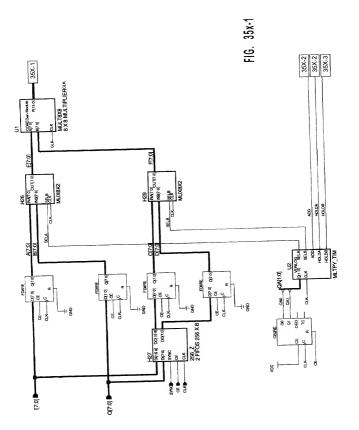
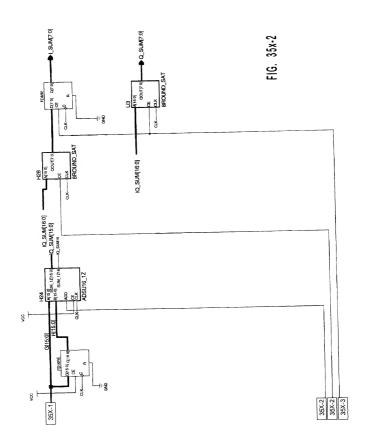
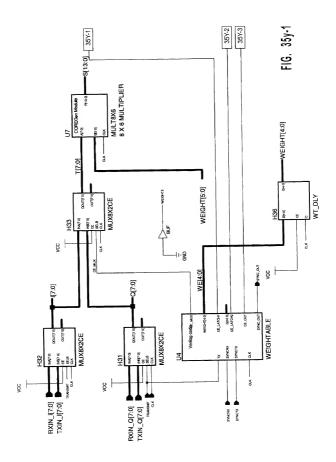


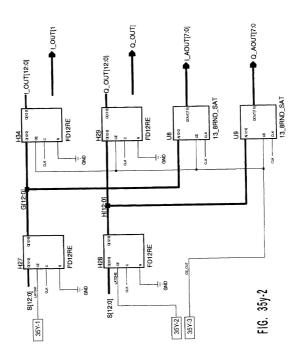
FIG. 35w











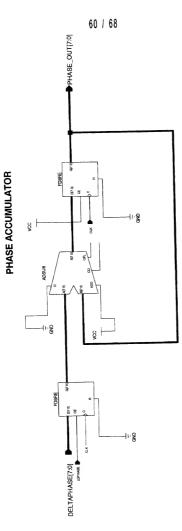


FIG. 35z

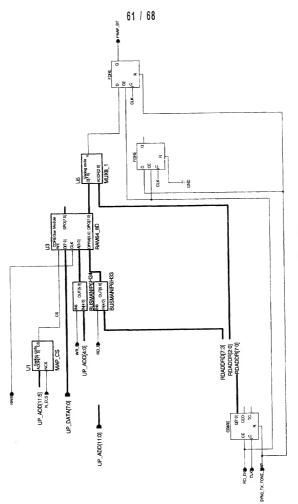
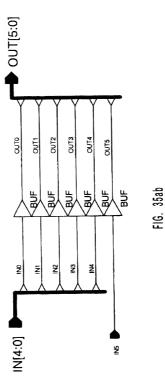


FIG. 35aa



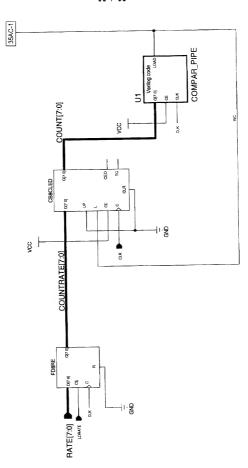


FIG. 35ac-1

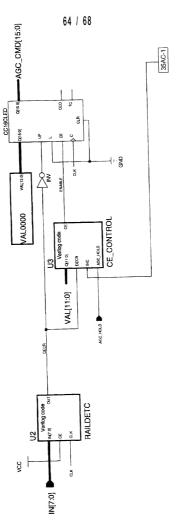


FIG. 35ac-2

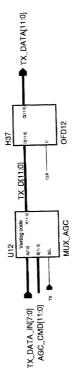
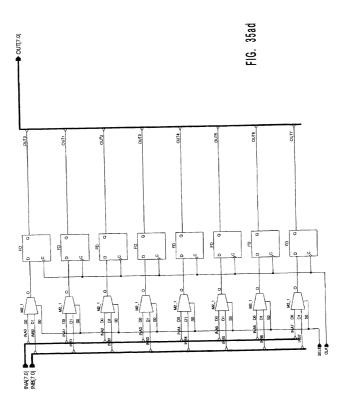


FIG. 35ac-3



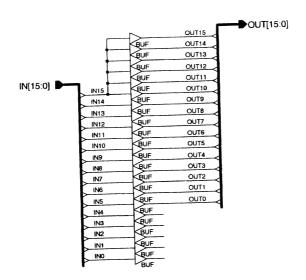


FIG. 35ae

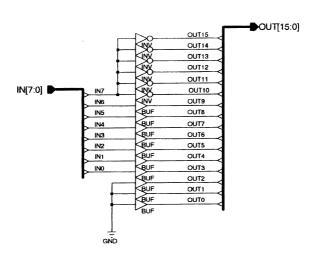


FIG. 35af